USING ALGEBRA (02/12/25)

Just two problems today, but they are quite tricky and require combining algebraic ideas in a non-trivial way. (So we'll get a chance to review these ideas.)

1. Let $f(x, y)$ be the real-valued function on a plane whose sum of values in the vertices of any regular 2025-gon equals zero. Prove that $f = 0$.

2. Prove that there exists a constant $C > 0$ such that for any graph Γ without loop or double edges, the following inequality holds:

$$T < C \cdot E^{3/2}.$$

Here T (resp. E) is the number of triangles in Γ (resp. number of edges).