POTLUCK WEEK

PIZZA CLUB

(1) **Pramana Saldin**: Let α be a rational number with $0 < \alpha < 1$ and $\cos(3\pi\alpha) + 2\cos(2\pi\alpha) = 0$. Prove that $\alpha = \frac{2}{3}$. (IMO Shortlist 1991)

(2) **Matthew Norton**: Let S be a class of functions from $[0, \infty)$ to $[0, \infty)$ that satisfies the following properties:

(i) The functions $f_1(x) = e^x - 1$ and $f_2(x) = \ln(x + 1)$ are in S;

(ii) If $f, g \in S$, then $f + g$ and $f \circ g$ are in S.

(iii) If $f, g \in S$ and $f(x) \geqslant g(x)$ for all $x \in [0, \infty)$, then $f - g$ is in S.

Prove that if $f, g \in S$, then the function $f \cdot g$ is also in S. (Putnam 2012 B1)

(3) **David Jiang**: Find the smallest n such that for all graphs where any vertex has degree at least n, there exists a cycle of even length. (Balkan Math Olympiad 2002)

(4) **Patrick Nylk**: The pizza club orders Glass Nickel's new pizza sphere for Wednesday's meeting. On this pizza sphere there are 5 point-sized pepperonis. Show that 4 of the 5 pepperonis lie on a closed pizza hemisphere. Is it always true that given $n+2$ point-sized pepperonis on S^{n-1}, $n+1$ pepperonis lie on a closed n-dimensional pizza hemisphere? (Putnam A2 2002)

(5) **Ansh Aggarwal**: What is the largest possible radius of a circle contained in a 4-dimensional hypercube of side length 1. (Putnam B3 2008)

(6) **Jonah Guse**: Let an unknown value b be fixed. Game show host Honte Mall places b dollars in one envelope and $2b$ dollars in another envelope. Honte then lets you choose one envelope and look inside. After this, you are given the opportunity to keep this envelope or exchange it for the other envelope. Either determine a strategy to win greater than $\frac{3b}{2}$ dollars on average or prove that no such strategy exists. (From *Elements of Information Theory* by Cover and Thomas)

(7) **Haran Mouli**: You are given 100 boxes, each containing a positive integer quantity of nuts, bolts, and washers. Prove that you can choose 51 of the boxes such that they contain strictly more than half of each object. (UW Math Talent Search 2023-24)

(8) **Ivan Hu**: Find all finite sequences $(x_0, \ldots, x_n)$ such that for every j, $0 \leqslant j \leqslant n$, x_j is equal to the number of times j appears in the sequence. (IMO Shortlist 2001)