

1. Find the limit of the sequence

$$0.2, 0.23, 0.233, 0.2333, \dots$$

2. Find the limit of the sequence

$$\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots$$

3. Find the limit:

$$\lim_{n \rightarrow \infty} \left( \frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \dots + \frac{n-1}{n^2} \right).$$

4. Find the limit:

$$\lim_{n \rightarrow \infty} \left( \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3} \right).$$

5. Find the limit:

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sqrt[n]{n(n+1)(n+2)\dots(n+n)}.$$

6. Find the limit:

$$\lim_{n \rightarrow \infty} \sum_{k=0}^{2n} \frac{k}{k+n^2}.$$

7. Find the number

$$x = \sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 4\sqrt{1 + 5\sqrt{1 + \dots}}}}}$$

8. Let  $x_0 = a$ ,  $x_1 = b$  and  $x_{n+2} = \frac{1}{3}(x_n + 2x_{n+1})$  for  $n = 0, 1, 2, \dots$ . Show that the limit  $\lim_{n \rightarrow \infty} x_n$  exists and find it.