Putnam Club Week Zero – 11 September 2012

Introduction

1. Prove that every (positive) composite integer n can be written as $n = xy+xz+yz+1$ for some positive integers x, y, and z. (Putnam 1988)

2. The number of distinct positive divisors of a positive integer n is a prime. Show that n is an integer power of a prime.

3. Given any five points on a sphere, show that some four of them must lie on a closed hemisphere. (Putnam 2002)

4. Show that $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ is never an integer for $n > 1$.

5. Let $p(x)$ be a polynomial with integer coefficients such that $p(0)$ and $p(1)$ are both odd. Show that $p(x)$ has no integer roots.

6. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous, and suppose that there is some real number a such that $f(f(f(a))) = a$. Show that there is some real number b such that $f(b) = b$.